Approximation Algorithms
Summer term 2009

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Exercise Set 2

Exercise 1:
Prove the NP-completeness of the following problems:

(i) INSTANCE: Natural numbers W and H and pairs $(w_i, h_i) \in \mathbb{N}^2$ for $1 \leq i \leq n$.
TASK: Is there a disjoint axis-parallel packing of the n rectangles with widths w_i and heights h_i inside a rectangle of width W and height H. Precisely, are there pairs $(x_i, y_i) \in \mathbb{N}^2$ for $1 \leq i \leq n$ such that

- $1 \leq i \leq n \rightarrow R_i \subseteq (0, W) \times (0, H)$
- $(x, y) \in \mathbb{R}^2 \rightarrow |\{i \,|\, (x, y) \in R_i\}| \leq 1$,

where $R_i := (x_i, x_i + w_i) \times (y_i, y_i + h_i)$.
Hint: Transformation from PARTITION

(ii) INSTANCE: Natural numbers $A, a_1, \ldots, a_n$.
TASK: Is there a disjoint axis-parallel packing of the n squares with side lengths a_i inside a rectangle with area A.

(4+4 Points)

Exercise 2:
Prove the NP-completeness of the following problems:

(i) INSTANCE: An undirected graph $G = (V, E)$ and an integer k.
TASK: Is there an $X \subseteq V$ with $|X| \leq k$ and $X \cup \Gamma(X) = V$?

(ii) INSTANCE: An undirected graph G.
TASK: Does G contain a Hamiltonian path?

(3+3 Points)

Exercise 3:
CLIQUE is NP-complete. Is it still NP-complete (provided that $P \neq NP$) if restricted to

(i) bipartite graphs,

(ii) planar graphs,

(iii) 2-connected graphs?

(2+2+3 points)

Please return the exercises until Tuesday, **April 28st, at 2:15 pm**.