Prof. Dr. S. Hougardy
Jan Schneider

Exercise Set 6

Exercise 1:
Consider the FRACTIONAL MULTIKNAPSACK problem: Given natural numbers m, n, w_i, c_{ij}, and W_j for $1 \leq i \leq n$ and $1 \leq j \leq m$, find $x_{ij} \in [0,1]$ satisfying $\sum_{j=1}^{m} x_{ij} = 1$ for all $1 \leq i \leq n$ and $\sum_{i=1}^{n} x_{ij} w_{ij} \leq W_j$ for all $1 \leq j \leq m$, such that $\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} c_{ij}$ is minimum. Provide a polynomial-time combinatorial algorithm for this problem or prove that it is NP-hard.

(4 Points)

Exercise 2:
Suppose that in an instance $a_1, \ldots, a_n$ of the BIN PACKING problem we have $a_i > \frac{1}{3}$ for $1 \leq i \leq n$.

(i) Reduce the problem to the CARDINALITY MATCHING problem.

(ii) Describe a linear-time algorithm that solves the problem.

(3+2 Points)

Exercise 3:
An algorithm for the BIN PACKING problem is called monotone if for inputs S and T with $S \subseteq T$ the algorithm needs at least as many bins for T as for S. Prove:

(i) Next Fit is monotone.

(ii) First Fit is not monotone.

(3+3 Points)

Please return the exercises until Tuesday, **May 26th, at 2:15 pm**.