Approximation Algorithms
Summer term 2009

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# Exercise Set 7

**Exercise 1:**
Consider the MULTIPROCESSOR SCHEDULING PROBLEM problem: Given a finite set $A$ of tasks, a positive number $t(a)$ for each $a \in A$ (the processing time), and a number $m$ of processors, find a partition $A = A_1 \cup \ldots \cup A_m$ of $A$ into $m$ disjoint sets such that $\max_{i=1}^{m} \sum_{a \in A_i} t(a)$ is minimum.

(i) Show that the problem is $NP$-hard.

(ii) Show that the greedy heuristic (subsequently assigning a task to the processor with the currently smallest load in any order) is a 2-approximation.

(iii) Show that for each fixed $m$ the problem has a fully polynomial-time approximation scheme.

(4+3+3 Points)

**Exercise 2:**
Let $k \geq 2$ be a fixed integer. The $k$-PARTITION problem is the following special case of BIN PACKING: Given $n = km$ integers $a_1, \ldots, a_n$, adding up to $mC$, and such that $\frac{C}{k+1} < a_i < \frac{C}{k-1}$ for all $i$, is there a partition of these numbers into $m$ groups of $k$ numbers, such that the sum in each group is precisely $C$. Show that 4-PARTITION is $NP$-complete.

(4 Points)

**Exercise 3*:**
Consider the problem of BIN PACKING where $a_i > c$ holds for $1 \leq i \leq n$ and some constant $c \geq 0$. Is there a $c < \frac{1}{3}$ such that the problem can be solved in polynomial time?

(4 Bonus Points)

Please return the exercises until Tuesday, **June 9nd, at 2:15 pm**.