Approximation Algorithms
Summer term 2009

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Exercise Set 10

Exercise 1:
Consider the relative greedy algorithm.

(i) For every $k \in \mathbb{N}$ describe an instance for which the relative greedy algorithm does not find an optimal solution.

(ii) What approximation guarantee does the algorithm have for $k = 5$?

(3+3 Points)

Exercise 2:
Denote by $\mathrm{SMT}^{\perp}(K)$ the length of a shortest rectilinear Steiner tree on the terminal set K and by $\mathrm{BB}(K)$ half of the circumference of the bounding box (i.e. $\mathrm{BB}(K) := \max_{(x,y)\in K} x - \min_{(x,y)\in K} x + \max_{(x,y)\in K} y - \min_{(x,y)\in K} y$). Show that $|K| \leq 4$ implies $\mathrm{SMT}^{\perp}(K) \leq \frac{3}{2} \cdot \mathrm{BB}(K)$

(4 Points)

Exercise 3:
The "vertex version" of the contraction lemma is wrong. Define a complete graph whose edge lengths fulfill the triangle inequality and vertex sets A, B, and C such that

$$0 < \mathrm{MST}(A) - \mathrm{MST}(A \cup C) < \mathrm{MST}(A \cup B) - \mathrm{MST}(A \cup B \cup C).$$

Here $\mathrm{MST}(X)$ for a vertex set X denotes the length of a minimum spanning tree in the graph induced by X.

(4 Points)

Please return the exercises until Tuesday, **June 30nd, at 2:15 pm**.