

Exercise Set 13

Exercise 1:

Let $PCP_\epsilon(r(n), q(n))$ be the same as $PCP(r(n), q(n))$ except that the constant $\frac{1}{4}$ is replaced by ϵ . Show that $NP \subseteq PCP_{n^{-\delta}}(\log^2 n, \log n)$ for every $\delta > 0$.

(3 Points)

Exercise 2:

Prove: $NP \subseteq coRP \Rightarrow NP \subseteq RP$

(4 Points)

Exercise 3:

Let k -MAXGSAT be the following problem: Given n variables x_1, \dots, x_n and a set of m boolean functions $\Phi = \{\Phi_1, \dots, \Phi_m\}$ using k variables each, find a truth assignment such that the number of satisfied functions is maximized. The goal is to show that there is no polynomial 2-approximation algorithm unless $P = NP$.

To do so, find a k and a polynomial reduction f from SAT to k -MAXGSAT with the following property: If I is a yes-instance of SAT, then all boolean functions in $f(I)$ are satisfiable. Otherwise, at most half of the functions in $f(I)$ are satisfiable.

Hint: Since $SAT \in NP$ there is a $(\log(n), 1)$ -verifier \mathcal{V} . Use \mathcal{V} to construct boolean formulas with the desired properties.

(5 Points)

Please return the exercises until Tuesday, **July 21nd, at 2:15 pm.**