Research Institute for Discrete Mathematics   Prof. Dr. S. Hougardy
Approximation Algorithms   J. Schneider
Summer term 2010

Exercise Set 3

Exercise 1:
Formulate fast 2-factor approximation algorithms for the following problems and prove
the approximation factor:

(i) Given an undirected graph $G = (V, E)$, what is the diameter of G? (The diameter
of G is defined as $\text{diam}(G) := \max_{v,w \in V} \text{dist}(v, w)$, where $\text{dist}(v, w)$ is the length
of a shortest v-w-path.)
Hint: Linear runtime is possible.

(ii) Given a directed graph with edge weights, find a directed acyclic subgraph of
maximum weight.

(4+4 Points)

Exercise 2:
Consider the following greedy algorithm for VERTEX COVER: Start with $C = \varnothing$. While
there are still edges in G, choose the node in G with the largest degree, add it to C, and
delete it from G. Show that the algorithm never produces a solution which costs more
than $\log n$ times the optimum.

(4 Points)

Exercise 3:
Show that any 4-colourable graph with n vertices can be coloured with $\mathcal{O}(n^{\frac{2}{3}})$ colours
in polynomial time.

(4 points)

Special topic:
From June 25th to 27th around 400 mathematicians from all over Germany will compete
in the German Football Championship of Mathematicians (Deutsche Fußballmeister-
schaft der Mathematiker) on Venusberg in Bonn. For this event organisers, assistants,
and players are needed. You can learn more about it at

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`http://www.dfmdm2010.de`

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and if you want to be part of it, send a mail to info@dfmdm2010.de.

Please return the exercises until Tuesday, **May 11th, at 2:15 pm**.