Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2010

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Exercise Set 4

Exercise 1:
Describe an algorithm which decides if a graph $G = (V, E)$ is 4-colourable with a running time of $\mathcal{O}(|E| \cdot 2^{|V|})$.

(4 points)

Exercise 2:
The optimization version of Exercise 2 (ii) from exercise set 2 is: Given an undirected graph $G = (V, E)$, find a set $X \subseteq V$ maximizing $|\delta(X)|$. Consider the following algorithm: Start with $X = \emptyset$. If adding a single vertex to X or deleting a single vertex from X makes $|\delta(X)|$ larger, then do so. Repeat until no improvement is possible.

(i) Show that the algorithm runs in polynomial time.

(ii) Show that this is a $\frac{1}{2}$-approximation algorithm.

(2+4 points)

Exercise 3:
Describe a $\frac{3}{4}$-approximation algorithm for KNAPSACK with running time $\mathcal{O}(n^3)$ that does not use the FPTAS from the lecture and give a proof of the approximation ratio.
Hint: The basic idea is to run, for every pair of items, a $\frac{1}{2}$-approximation algorithm on the remaining elements.

(5 points)

Special topic:
The next meeting of the institute's group of mentors takes place Tuesday, May 18th, at 6:00 pm in the conference room of the Arithmeum. The topic is "Data Structures: AVL Trees" and all interested students are invited.

Please return the exercises until Tuesday, **May 18th, at 2:15 pm**.