Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2010

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# Exercise Set 5

**Exercise 1:**
Describe exact algorithms with running times of $\mathcal{O}(2^{\frac{n}{2}})$ for the following problems:

(i) Subset Sum, where $n$ is the number of numbers.

(ii) Knapsack, where $n$ is the number of items.

(3+3 points)

**Exercise 2:**
An algorithm for the Bin Packing problem is called monotone if for inputs $S$ and $T$ where $S$ is a subsequence of $T$ the algorithm needs at least as many bins for $T$ as for $S$. Prove:

(i) The Next-Fit algorithm is monotone.

(ii) The First-Fit algorithm is not monotone.

(3+3 points)

**Exercise 3:**
Maximum Clique is the problem of finding the maximum number of vertices in a complete subgraph. Given an undirected graph $G = (V, E)$ and some integer $k \geq 1$, we define $G^{(k)}$ to be the undirected graph $(V^{(k)}, E^{(k)})$, where $V^{(k)}$ is the set of all $k$-tuples of vertices from $V$ and $\{(v_1, \dots, v_k), (w_1, \dots, w_k)\} \in E^{(k)}$ if and only if for each $i$ (with $1 \leq i \leq k$) either $\{v_i, w_i\} \in E$ or $v_i = w_i$ holds. Prove:

(i) If $\omega(G)$ denotes the size of a maximum clique in a graph $G$, then $\omega(G)^k = \omega(G^{(k)})$.

(ii) If there is an approximation algorithm for Maximum Clique with a constant approximation ratio, then there is an FPTAS for the problem.

(3+3 points)

Please return the exercises until Tuesday, **June 1st, at 2:15 pm**.