Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2010

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Exercise Set 7

Exercise 1:
An *approximation algorithm with absolute error* B is an algorithm A such that $|A(I) - OPT| \leq B$ holds for any instance I.

(i) Prove: If $P \neq NP$, then for all $B \in \mathbb{N}$ there is no approximation algorithm with absolute error B for the STEINER TREE PROBLEM.

(ii) A construction similar to the one in (i) works for many problems, but there are also NP-hard problems which have approximation algorithms with absolute error 1. Find an NP-hard optimization variant of PARTITION for which such an algorithm exists (and prove that it exists).

(4+4 points)

Exercise 2:
Consider the STEINER TREE PROBLEM on a network $N = (V, E, c, K)$ where (V, E) is the underlying graph, $c : E \to \mathbb{R}_+$ describes the lengths of the edges, and $K \subseteq V$ is the terminal set. Let the *complete distance network* $N_D = (K, E_D, c_D)$ be a complete graph (K, E_D) on the terminal set with edge lengths $c_D : E_D \to \mathbb{R}_+$ defined as $c_D(\{v, w\}) := \mathrm{dist}_N(v, w)$, where $\mathrm{dist}_N(v, w)$ is the length of a shortest path from v to w in N.

(i) Prove: If T is a minimum spanning tree in N_D, then $c_D(T) \leq (2 - \frac{2}{|K|}) \cdot smt(N)$, where $smt(N)$ is the length of a minimum Steiner tree in N. (It follows that the algorithm of Kou, Markowsky, and Berman is a $(2 - \frac{2}{|K|})$-approximation.)

(ii) Show that the bound in (i) is best possible.

(4+3 points)

Special topic:
The next meeting of the institute's group of mentors takes place on Tuesday, June 15$^{\text{th}}$, at 6:00 pm in the conference room of the Arithmeum. The topic is “Hypergraphs Reloaded” and all interested students are invited.

Please return the exercises until Tuesday, **June 15th, at 2:15 pm**.