Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2010

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Exercise Set 9

Exercise 1:
Consider the following algorithm for the STEINER TREE PROBLEM with 3 terminals v_1, v_2, and v_3: Find the shortest path P between v_1 and v_2 and let a be the distance of v_3 to P. Then find a vertex z which minimizes $\sum_{i=1}^{3} dist(v_i, z)$ under the conditions $\mathrm{dist}(v_i, z) \leq \mathrm{dist}(v_1, v_2)$ for $i \in \{1, 2\}$ and $\mathrm{dist}(v_3, z) \leq a$. The algorithm finally returns the union of the shortest paths from z to the terminals. Show that the algorithm can be implemented in $\mathcal{O}(m+n \log n)$ and works correctly. (You can use the fact that Dijkstra's algorithm can be implemented in $\mathcal{O}(m + n \log n)$.)

(4 points)

Exercise 2:
Consider the Relative Greedy algorithm.

(i) For each $k \in \mathbb{N}$, $k > 2$, find an instance of the STEINER TREE PROBLEM for which the solution found by the algorithm is not optimal.

(ii) What approximation factor does the algorithm have for $k = 5$?

(4+2 points)

Exercise 3:
Show that the contraction lemma still holds if one adds edges between terminals whose lengths are larger than 0. ("Adding" an edge means that if there already is an edge, a parallel edge is inserted.)

(4 points)

Exercise 4:
Compute a shortest rectilinear Steiner tree on the terminal set $\{(4, 5), (1, 4), (2, 1), (4, 2)\}$. You need to justify your solution.

(2 points)

Special topic:
As a reminder, this evening Johannes Zühlke from the Fraunhofer-Institut für Algorithmen und wissenschaftliches Rechnen will give the talk "Diskrete Mathematik jenseits der Uni" (6:00 pm in the Gerhard-Konow-Hörsaal).

Please return the exercises until Tuesday, **June 29th, at 2:15 pm**.