Research Institute for Discrete Mathematics
Approximation Algorithms
Summer term 2010

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Exercise Set 10

Exercise 1:
A weight function c on a bipartite graph G satisfies the *square inequality* if $c(a,b) + c(a',b) + c(a',b') \geq c(a,b')$ for all $\{a,b\}, \{a',b\}, \{a,b'\}, \{a',b'\} \in E(G)$. An instance of the Bipartite TSP with Square Inequality is a complete bipartite graph with non-negative edge weights. Show that if there is an α-approximation algorithm for the Bipartite TSP with Square Inequality, then there also is an α-approximation algorithm for the Metric TSP. (Hint: Given an instance (G,c) of the Metric TSP, construct an instance (H,d) of the other problem with $V(H) := V(G) \times \{1,2\}$ and $d(\{(v,1),(w,2)\}) \in \{c(\{v,w\}), 0\}$.)

(4 points)

Exercise 2:
For a graph with weighted edges, the Traveling Salesmen Problem is the problem of finding a collection of cycles covering all vertices that have a minimum weight. The weight of such a collection is the sum of weights of the cycles's edges. Show that the Traveling Salesmen Problem can be solved in polynomial time. (You can use that a perfect matching can be found in polynomial time.)

(3 points)

Exercise 3:
Let c_0 be the value of an optimal solution of an instance of the Metric TSP and c_1 the cost of a second-shortest tour. Prove $\frac{c_1 - c_0}{c_0} \leq \frac{2}{n}$.

(4 points)

Exercise 4:
Show that the 2-OPT algorithm has an approximation guarantee of $\frac{3}{2}$ on graphs which have edge lengths 1 or 2.

(4 points)

Please return the exercises until Tuesday, **July 6th, at 2:15 pm**.