Prof. Dr. S. Hougardy
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Exercise Set 12

Exercise 1:
Given an instance (K_n, c) of the TRAVELING SALESMAN PROBLEM and an edge $e \in E(K_n)$, show that it is NP-complete to decide whether e occurs in an optimal tour.

(5 points)

Exercise 2:
Why does $PCP(0, \text{poly}(n)) = NP$ hold with $\text{poly}(n) := \bigcup_{k \geq 0} n^k$?

(3 points)

Exercise 3:
Consider the following problem: Given n variables $x_1, \ldots, x_n$ and a set of m boolean functions $\Phi = \{\Phi_1, \ldots, \Phi_m\}$ using k variables each, find a truth assignment such that the number of satisfied functions is maximized. The goal is to show that there is no polynomial-time 2-approximation algorithm unless $P = NP$.
To do so, find a polynomial reduction f from SAT to the problem with the following property: If I is a yes-instance of SAT, then all boolean functions in $f(I)$ are satisfiable. Otherwise, at most half of the functions in $f(I)$ are satisfiable. (Hint: Use that there is a $(log(n), 1)$-verifier $\mathcal{V}$ for SAT.)

(5 points)

Please return the exercises until Tuesday, **July 20th, at 2:15 pm**.