Research Institute for Discrete Mathematics
Chip Design
Summer term 2013

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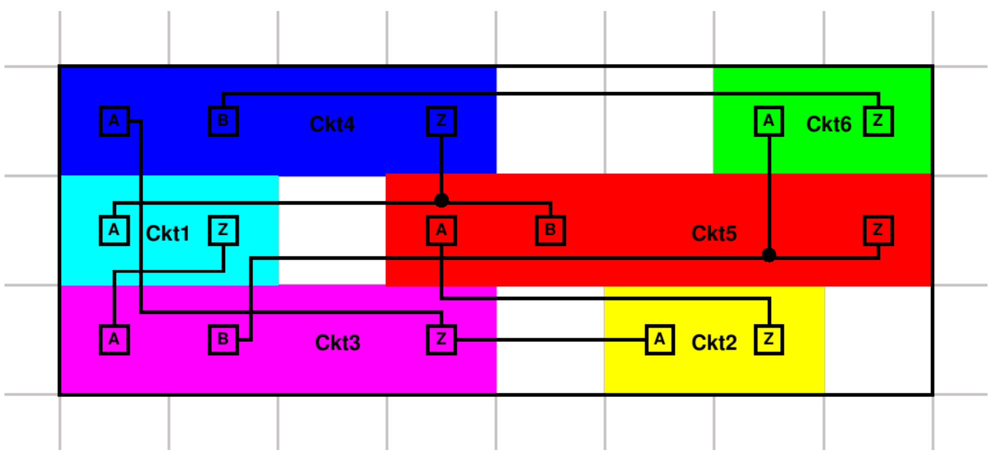
Exercise Set 2

Exercise 2.1:
Consider the placement instance in the figure below. Each of the circuits Ckt1, Ckt2, Ckt3, Ckt4, Ckt5, and Ckt6 must be placed in one of the three circuit rows. Of course, their placed areas must also be within the chip area (black outline) and must not intersect each other. The orientation of the circuits must not be changed. The figure shows a feasible placement.
Pins (can be seen as point-shaped objects) are marked by small squares within their circuits. The centers of these squares are the pin locations. They are fixed relative to their circuit. Nets are described by Steiner trees connecting their pins. These Steiner trees are not pairwise disjoint; therefore Steiner points are drawn as filled circles. Two adjacent parallel grey lines have distance one.



The *bounding box net length* of a placement is defined as

$$\mathrm{BB}(\mathcal{N}) := \sum_{N \in \mathcal{N}} \mathrm{BB}(N)$$

where $\mathcal{N}$ is the set of nets in our instance.
The placement in the figure has a bounding box net length of 32.

(a) Prove that there is no feasible placement with $\mathrm{BB}(\mathcal{N}) < 9$. Can you find a better lower bound? (2 points)

(b) Determine a feasible placement of minimum bounding box net length. (4 points)

Exercise 2.2:
Prove that the following problem is *NP*-complete for each constant $\alpha \geq 1$:

> **Input:** A set $\mathcal{C} = \{[0, w_i] \times [0, h_i] \mid i = 1, \ldots, n\}$ of rectangular circuits and a rectangular chip area $\square = [0, w] \times [0, h]$ such that $\alpha \cdot \sum_{i=1}^{n} w_i h_i \leq wh$.
>
> **Question:** Is there a feasible placement?

(5 points)

Exercise 2.3:
Prove that the following problem is *NP*-complete:

> **Input:** An undirected graph $G = (V, E)$, edge weights $w : E(G) \to \mathbb{Z}^+$ and integers $C, D \in \mathbb{N}$.
>
> **Question:** Is there a spanning tree T in G such that

$$\sum_{e \in E(T)} w(e) \leq C \quad \text{and} \quad \sum_{e \in E(P)} w(e) \leq D \text{ for all paths } P \text{ in } T?$$

Hint: Reduce 3-Sat to the above problem. Include a vertex for each literal and for each clause as well as a "true-setting" node and a root node in which each maximal path starts. How can you model assignment (setting literals to true or false), consistency (contradicting literals mustn't have the same assignment) and containment of literals in clauses? How can you guarantee that each spanning tree of an unsatisfiable instance has high cost or has a long path with high cost?

(5 points)

Deadline: Thursday, April 25th, before the lecture.