Exercise Set 7

Exercise 7.1. Show that the family of bridge sets, $\mathcal{B}_W(R)$, is the set of bases of a matroid for any tree (T, W) and any $R \subseteq T$.

(5 points)

Exercise 7.2. Consider the POINT-TO-POINT CONNECTION PROBLEM: Given an undirected graph G with edge weights $c : E(G) \to \mathbb{R}_+$ and sets $S, T \subseteq V(G)$ with $S \cap T = \emptyset$ and $|S| = |T| \geq 1$, find a set $F \subseteq E(G)$ of minimum cost such that there is a bijection $\pi : S \to T$ and paths from s to $\pi(s)$ for all $s \in S$ in $(V(G), F)$.

Show that $f(X) = 1$ if $|X \cap S| \neq |X \cap T|$ and $f(X) = 0$ otherwise defines a proper function.

(5 points)

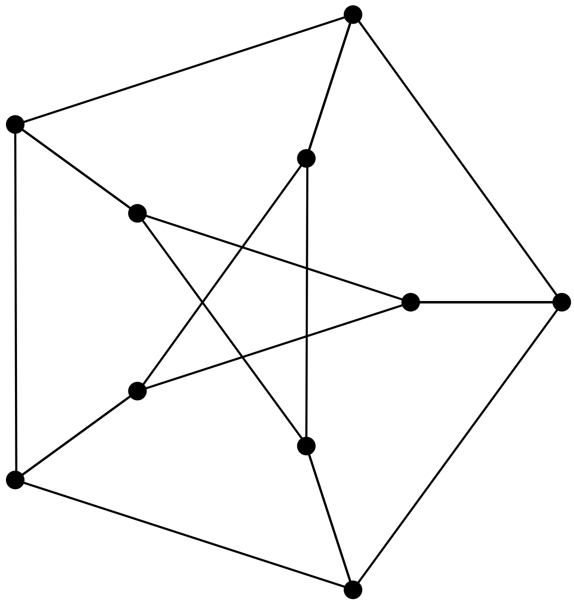
Exercise 7.3. Show that the clean-up step of the PRIMAL-DUAL ALGORITHM FOR NETWORK DESIGN (the step that (possibly) removes edges from F) is crucial: without this step, the algorithm does not even achieve any finite performance ratio for $k = 1$.

(5 points)

Exercise 7.4.

$$\begin{aligned} &\min \sum_{e \in E(G)} x_e \\ &\text{s.t.} \sum_{e \in \delta(S)} x_e \geq f(S) \quad (S \subseteq V(G)) \\ &\qquad\quad x_e \geq 0 \qquad\quad (e \in E(G)) \end{aligned} \tag{1}$$

Find an optimum basic solution x for the linear program (1), where G is the Petersen graph (see figure below) and $f(S) = 1$ for all $\emptyset \neq S \subsetneq V(G)$. Find a maximal laminar family $\mathcal{B}$ of tight sets with respect to x such that the vectors $\chi^{\mathcal{B}}$, $B \in \mathcal{B}$, are linearly independent.



(5 points)

Deadline: Thursday, June 14th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss18/appr_ss18_ex.html`

In case of any questions feel free to contact me at traub@or.uni-bonn.de.