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Exercise Set 7

Exercise 7.1. Provide a polynomial time algorithm for the Standard Placement Problem restricted to instances with only one circuit.

(5 points)

Exercise 7.2. The Gridded Placement Problem is an extension of the Standard Placement Problem with a grid $\Gamma = \Gamma_x \times \Gamma_y$ where $\Gamma_z := \{k \cdot \delta_z : k \in \mathbb{Z}\}$ with $\delta_z \in \mathbb{Z}$ for $z \in \{x, y\}$. In this variant, the lower left corner of each circuit is required to be in Γ.

Prove that the Gridded Placement Problem is NP-hard even if an optimum solution of the associated ungridded placement problem is known.

(5 points)

Exercise 7.3. Prove that unless P = NP, there is no polynomial time n^α approximation algorithm for the Quadratic Assignment Problem for any $\alpha < 1$ even if $w \equiv 1$, $c \equiv 0$, d is metric and G is a tree.

(5 points)

Exercise 7.4. Let $G = (V, E)$ be an undirected graph with edge weights $w : E \to \mathbb{R}_{\geq 0}$ and $k \in \mathbb{N}$. Let $C \subseteq V$ and $f : V \setminus C \to \{1, \dots, k\}$ be a placement function. We are looking for positions $f : C \to \{1, \dots, k\}$ s.t.

$$\sum_{e=\{v,w\} \in E} w(e) \cdot |f(v) - f(w)|$$

is minimum. Note that f is not required to be injective.

Prove that this problem can be solved optimally by solving $k - 1$ minimum weight s-t-cut problems in digraphs with $\mathcal{O}(|V|)$ vertices and $\mathcal{O}(|E|)$ edges.

Hint: Consider digraphs $G_j = (V_j, E_j)$ with $V_j := \{s, t\} \cup C$ and

$$\begin{aligned} E_j := &\Big\{(s, v) : \exists\, w \in V \setminus C, f(w) \leq j, \{v, w\} \in E\Big\} \cup \\ &\Big\{(v, w) : v, w \in C, \{v, w\} \in E\Big\} \cup \\ &\Big\{(v, t) : \exists\, w \in V \setminus C, f(w) > j, \{v, w\} \in E\Big\} \end{aligned}$$

(5 points)

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Deadline: June 12th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss18/chipss18.html`

In case of any questions feel free to contact me at bihler@or.uni-bonn.de.