Exercise Set 8

Exercise 8.1. Consider the following variant of the Single Row Placement With Fixed Ordering problem, in which we minimize the bounding box net length:

> **Input:** A set $\mathcal{C} = \{C_1, \ldots, C_n\}$ of circuits, widths $w(C_i) \in \mathbb{R}_+$, an interval $[0, w(\square)]$, s.t. $\sum_{i=1}^{n} w(C_i) \leq w(\square)$. A netlist $(\mathcal{C}, P, \gamma, \mathcal{N})$ where the offset of a pin $p \in P$ satisfies $x(p) \in [0, w(\gamma(p))]$. Weights $\alpha : \mathcal{N} \to \mathbb{R}_+$.
>
> **Task:** Find a feasible placement given by a function $x : \mathcal{C} \to \mathbb{R}$ s.t. $0 \leq x(C_1)$, $x(C_i) + w(C_i) \leq x(C_{i+1})$ for $i = 1, \ldots, n-1$ and $x(C_n) + w(C_n) \leq w(\square)$, that minimizes
>
> $$\sum_{N \in \mathcal{N}} \alpha(N) \cdot \mathrm{BB}(N).$$

Here, $\mathrm{BB}(N)$ denotes the bounding box net length.

Show that there exist $f_i : [0, w(\square)] \to \mathbb{R}$, $i = 1, \ldots, n$, piecewise linear, continuous and convex, such that we can solve this problem by means of the Single Row Algorithm.

(5 points)

Exercise 8.2. Formulate the Simple Global Routing Problem as an integer linear program with a polynomial number of variables and constraints.

(5 points)

Exercise 8.3. Consider quadratic netlength minimization in x-dimension based on the (quadratic) Clique netmodel i.e.

$$\text{CliqueSQ}(N) := \sum_{\{p,q\} \subseteq N} \frac{w(N)}{|N| - 1} \Big(x(p) + x(\gamma(p)) - x(q) - x(\gamma(q)) \Big)^2$$

(a) Show that CliqueSQ can be replaced equivalently by the quadratic StarSQ netmodel

$$\text{StarSQ}(N) := w'(N) \cdot \min \left\{ \sum_{p \in N} (x(p) + x(\gamma(p)) - c)^2 \, | c \in \mathbb{R} \right\}$$

for an appropriate weight function w'.

(b) For a fixed placement x and a single net N let $l, r \in N$ be defined as $l := \arg\min\{x(p) + x(\gamma(p)) \mid p \in N\}$ and $r := \arg\max\{x(p) + x(\gamma(p)) \mid p \in N\}$. We further define for $p, q \in N$

$$w_{pq}^{\text{B2B}} := \begin{cases} 0 & \text{if } \{p,q\} \cap \{l,r\} = \emptyset, \\ \left| x(q) + x(\gamma(q)) - x(p) - x(\gamma(p)) \right|^{-1} & \text{else.} \end{cases}$$

Show that the CliqueSQ netlength with weights w^{B2B} equals the (linear) bounding box netlength for placement x.

(3 + 3 points)

Exercise 8.4. Consider following specialized legalization problem. A feasible placement consists of integral placement coordinates $x, y : \mathcal{C} \to \mathbb{Z}^2$ (such that all circuits are located within the chip image), and all circuits have unit height and width. In addition, there is an (infeasible) input placement $\tilde{x}, \tilde{y} : \mathcal{C} \to \mathbb{R}^2$.

- Find a polynomial-time algorithm that finds a legal placement minimizing the linear or quadratic movement.
- Find a linear time algorithm that finds a legal placement minimizing the quadratic movement if the instance consists of a single row.

(2+2 points)

Deadline: June 19th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss18/chipss18.html`

In case of any questions feel free to contact me at bihler@or.uni-bonn.de.