Exercise Set 10

Exercise 10.1. Let $\alpha > 1$ and $1 \leq \beta < 1+2/(\alpha-1)$. Construct a connected, planar graph G with $w : E(G) \to \mathbb{R}_+$ and $r \in V(G)$ that contains no spanning tree T with the following properties:

(a) For each $v \in V(G)$: $\text{dist}_{w,T}(r,v) \leq \alpha \cdot \text{dist}_{w,G}(r,v)$.
(b) For a minimum-spanning tree M: $\sum_{e \in E(T)} w(e) \leq \beta \cdot \sum_{e \in E(M)} w(e)$.

(5 points)

Exercise 10.2. Given a root $r \in \mathbb{R}^2$, a finite set of sinks $S \subset \mathbb{R}^2$, Lagrangean multipliers $(\lambda_s)_{s \in S}$, the rectilinear cost-distance Steiner arborescence problem asks for a Steiner arborescence Y rooted at r, minimizing

$$\sum_{(v,w) \in E(Y)} ||v-w||_1 + \sum_{s \in S} \lambda_s \cdot \left(\sum_{(v,w) \in E(Y_{[r,s]})} ||v-w||_1 \right)$$

Using the light-approximate shortest path tree algorithm, approximate this problem up to a factor of 3 in $\mathcal{O}(n \log n)$.

(5 points)

Exercise 10.3. A posynomial function $f : \mathbb{R}^n_{>0} \to \mathbb{R}$ is of the form

$$f(x) = \sum_{k=1}^{K} c_k \prod_{i=1}^{n} x_i^{a_{ik}}$$

for $K \in \mathbb{N}, c_k > 0$ and $a_{ik} \in \mathbb{R}$.

(a) Give an example for a non-convex posynomial function.

(b) Let f be a posynomial function with lower and upper bounds $l, u \in \mathbb{R}^n_{>0}$, $l \leq u$ on the variables. Show that each local minimum of f on the box $[l,u]$ is also a global minimum of f on $[l,u]$.

Hint: Use a logarithmic variable transformation to derive an equivalent convex problem.

(2 + 3 points)

Exercise 10.4. Consider a chain of $n \in \mathbb{N}$ continuously sizable inverters with sizes $x_i > 0$ $(1 \leq i \leq n)$ depicted in Figure 10.1. Assume that the delay

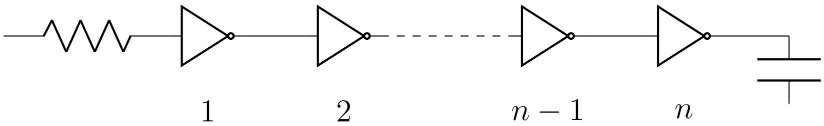

Figure 10.1: Chain of inverters.

θ_i through inverter i is given by

$$\theta_i(x) = \alpha + \frac{\beta \cdot x_{i+1}}{x_i} \qquad \text{for } 1 \leq i < n-1$$

where $x = (x_1, \ldots, x_n)$, $\alpha \geq 0$, $\beta > 0$. Wire delays, slews and transitions are ignored.

Derive a closed formula for the size x_i of the i-th inverter in a solution x of the total delay minimization problem for fixed x_1, x_n:

$$\min \left\{ \sum_{i=1}^{n-1} \theta_i(x) : x_i > 0 \text{ for all } 2 \leq i \leq n-1 \right\}.$$

(5 points)

Deadline: July 10th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss18/chipss18.html`

In case of any questions feel free to contact me at bihler@or.uni-bonn.de.