Exercise Set 4

Exercise 4.1. Describe an algorithm which decides if an undirected graph $G = (V, E)$ is 4-colorable in time $\mathcal{O}(|E| \cdot 2^{|V|})$.

(3 points)

Exercise 4.2. Let G be a k-colorable graph with n vertices, where k is a constant. We define $x_k := n^{1-\frac{1}{k-1}}$ and for $2 \leq l < k$, $x_l := x_{l+1}^{1-\frac{1}{l-1}}$. For simplicity, we assume that n is chosen such that x_l is a natural number for $l \in \{2, \ldots, k\}$.

Prove that there exists a polynomial time algorithm that colors G with kx_k colors.

(6 points)

Exercise 4.3. Consider the following procedure for (unweighted) Minimum Vertex Cover: Given a graph G, compute a DFS tree for every connected component. Return all vertices with non-zero out-degree in the tree. Show that this is a 2-approximation algorithm.

(3 points)

Exercise 4.4. Consider the following variant of Set Cover:

Instance: A set U, sets $\mathcal{S} = \{S_1, \ldots, S_m\}$ such that $\bigcup_{S \in \mathcal{S}} S = U$, an integer $k \in \mathbb{N}$.

Output: k sets $S_{i_1}, \ldots, S_{i_k} \in \mathcal{S}$ such that $\left| \bigcup_{j=1}^{k} S_{i_j} \right|$ is maximum.

Show that iteratively picking the element that maximizes the amount of not yet covered elements is a $(1 - \frac{1}{e})$-approximation.

(4 points)

Deadline: Thursday, May 2nd, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/appr_ss19_ex.html`

In case of any questions feel free to contact me at rockel@or.uni-bonn.de.