Prof. Dr. Stefan Hougardy
Benjamin Rockel

Exercise Set 5

Exercise 5.1. The LP relaxation of the Minimum Weight Vertex Cover Problem is

$$\min\{cx : M^T x \geq 1, x \geq 0\}$$

where $M \in \{0,1\}^{n \times m}$ is the incidence matrix of an undirected graph G and $c \in \mathbb{R}_+^{V(G)}$. A *half-integral* solution for this relaxation is one with entries 0, $\frac{1}{2}$ and 1 only.

(i) Show that the above LP relaxation of the Minimum Weight Vertex Cover Problem always has a half-integral optimum solution.

(ii) Use this to obtain a 2-approximation algorithm.

(iii) Given a graph $G = (V, E)$ with weights $c \in \mathbb{R}_+^n$ and a coloring $\varphi : V \to \{1, \ldots, k\}$, show how the LP relaxation can be used to find a vertex cover $X \subseteq V$ with cost $c(X) \leq (2 - \frac{2}{k})\text{OPT}$. Here OPT denotes the cost of an optimum solution.

(4+1+3 points)

Exercise 5.2. Consider the Fractional Multi Knapsack Problem: Given natural numbers $n, m \in \mathbb{N}$ and w_i, $c_{ij} \in \mathbb{N}$ as well as $W_j \in \mathbb{N}$ for $1 \leq i \leq n$ and $1 \leq j \leq m$, find $x_{ij} \geq 0$ satisfying $\sum_{j=1}^m x_{ij} = 1$ for all $1 \leq i \leq n$ and $\sum_{i=1}^n x_{ij} w_i \leq W_j$ for all $1 \leq j \leq m$ such that $\sum_{i=1}^n \sum_{j=1}^m x_{ij} c_{ij}$ is minimum.

State a polynomial-time combinatorial algorithm for this problem.
(Do not use that a linear program can be solved in polynomial time.)

(4 points)

Exercise 5.3. Show that the following variant of the Knapsack Problem is NP-hard:

$$\max\left\{\sum_{i=1}^n c_i x_i \,:\, \sum_{i=1}^n w_i x_i \leq W,\, x_i \in \mathbb{Z}_{\geq 0}\, \forall\, 1 \leq i \leq n\right\} \tag{1}$$

(Here, we allow to use an item several times.) You may use that the Knapsack Problem is NP-hard.

(4 points)

Information: The student council of mathematics will organize the math party on 9/05 in N8schicht. The presale will be held on Mon 6/05, Tue 7/05 and Wed 8/05 in the mensa Poppelsdorf. Further information can be found at fsmath.uni-bonn.de.

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Deadline: Thursday, May 9th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/appr_ss19_ex.html`

In case of any questions feel free to contact me at rockel@or.uni-bonn.de.