Summer term 2022
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Linear and Integer Optimization
Assignment Sheet 5
Inofficial English Translation

1. Let P be a polyhedron and F a face of P. Show that

$$\{c \mid c^t z = \max\{c^t x \mid x \in P\} \text{ for all } z \in F\}$$

is a polyhedral cone. (5 points)

2. Let $P = \{x \in \mathbb{R}^n \mid Ax = b, x \geq 0\}$ be a polyhedron with $\text{rank}(A) = m < n$. Show: A vector $x' \in P$ is a vertex of P if and only if it is a feasible basic solution. (5 points)

3. Consider the following linear system of inequalities:

$$\begin{aligned} x_1 + x_2 &\leq 6 \\ x_2 &\leq 3 \\ x_1 + 2x_2 &\leq 9 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Sketch the solution space, transform the system by adding slack variables x_3, x_4, x_5 to the form $Ax = b, x \geq 0$, and determine all bases and the corresponding basic solutions. Are there any degenerate or infeasible basic solutions? If so, which ones? (2 points)

p.t.o.

4. Use the SIMPLEX ALGORITHM to solve the following linear programs:

(a)

$$\begin{array}{rrcl} & \max\ 2x_2 & & \\ \text{s.t.} & x_1 - x_2 & \leq & 4 \\ & -x_1 + x_2 & \leq & 1 \\ & x_1, x_2 & \geq & 0 \end{array}$$

(b)

$$\begin{array}{rrcl} & \max\ 5x_1 + 3x_2 & & \\ \text{s.t.} & 4x_1 + 2x_2 & \leq & 12 \\ & 4x_1 + x_2 & \leq & 10 \\ & x_1 + x_2 & \leq & 4 \\ & x_1, x_2 & \geq & 0 \end{array}$$

(c)

$$\begin{array}{rrcl} & \min\ 5x_1 - x_2 & & \\ \text{s.t.} & x_1 - 3x_2 & \leq & 1 \\ & x_1 - 4x_2 & \leq & 3 \\ & x_1, x_2 & \geq & 0 \end{array}$$

(d)

$$\begin{array}{rrcl} & \min\ -x_1 - 2x_2 & & \\ \text{s.t.} & 2x_1 + x_2 & \leq & 5 \\ & -x_1 - x_2 & \geq & -3 \\ & x_1, x_2 & \geq & 0 \end{array}$$

Show all intermediate simplex tableaus and describe why you can choose a specific variable to enter or leave the basis. If there is an optimum solution, also give its value. (2+2+2+2 points)

Due date: Thursday, May 12, 2022, before the lecture in the lecture hall.