Summer term 2022
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Linear and Integer Optimization
Assignment Sheet 7
Inofficial English Translation

1. Show for $A \in \mathbb{Q}^{n\times n}$ the following statements:
   (a) $\text{size}(\det(A)) \le 2\text{size}(A)$.
   (b) If A is regular then $\text{size}(A^{-1}) \le 4n^2\text{size}(A)$. (2+1 points)

2. Let $A := \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ s & -1 \end{pmatrix}$ and $b := \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.
   Use the IDEALIZED ELLIPSOID ALGORITHM with $R = 2$ to compute a feasible solution in $P = \{x \in \mathbb{R}^2 \mid Ax \le b\}$ for $s = -1$ and for $s = -2$. (4 points)

3. Define $\|A\| := \max_{\|x\|=1} \|Ax\|$ for $A \in \mathbb{R}^{n\times n}$, where $\|\cdot\| : \mathbb{R}^n \to \mathbb{R}$ is the standard Euclidean norm. Prove:
   (a) $\|A\|$ is a norm
   (b) $\|aa^t\| = a^t a$
   (c) $\|A\| = \max\{x^t Ax \mid \|x\| = 1\}$ if A is positive semidefinite
   (d) $\|A\| \le \|A + B\|$ if A and B are positiv semidefinite. (1+2+2+1 points)

4. Show that $|\det(A)| \le \prod_{i=1}^n \|a_i\|$ for an $n\times n$-matrix A with columns $a_1, \ldots, a_n$ (where $\|\cdot\| : \mathbb{R}^n \to \mathbb{R}$ is again the standard Euclidean norm). (2 points)

Due date: Tuesday, May 24, 2022, before the lecture in the lecture hall.