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Exercises 7

Exercise 1:
Show that a 2-factor approximation for a (cardinality maximal) b-matching in a graph can be found in linear time.

(4 points)

Exercise 2:
Let G be a k-regular and $(k-1)$-edge-connected graph with an even number of vertices, and let $c : E(G) \to \mathbb{R}_+$. Prove that there exists a perfect matching M in G with $c(M) \geq \frac{1}{k} c(E(G))$.
Hint: Show that $\frac{1}{k}\mathbb{1}$ is in the perfect matching polytope, where $\mathbb{1}$ denotes a vector whose components are all one.

(4 points)

Exercise 3:
Show that a minimum weight perfect simple 2-matching in an undirected graph G can be found in $O(n^6)$ time.

(4 points)

Exercise 4:
Let G be a graph, and let

$$P \;:=\; \left\{ x \in \mathbb{R}_+^{|E(G)|} \;:\; \sum_{e \in \delta(v)} x_e = 1 \quad \text{for all } v \in V(G) \right\}$$

be the *fractional perfect matching polytope* of G. Prove that the vertices of P are exactly the vectors x with

$$x_e = \begin{cases} \frac{1}{2} & \text{if } e \in E(C_1) \cup \ldots \cup E(C_k) \\ 1 & \text{if } e \in M \\ 0 & \text{otherwise} \end{cases},$$

where $C_1, \ldots, C_k$ are vertex-disjoint odd circuits and M is a perfect matching in $G - (V(C_1) \cup \ldots \cup V(C_k))$.

(4 points)

Deadline: Tuesday, November 30th, before the lecture.