Research Institute for Discrete Mathematics
University of Bonn

Linear and Integer Optimization
Assignment Sheet 1

1. A paper mill produces paper rolls of 3 m width. The customers order rolls with smaller widths and the mill has to cut the ordered rolls out of the 3 m wide rolls. For example, a 3 m wide roll may be cut into two 93 cm wide and a 108 cm wide roll, leaving an offcut of 6 cm. The current order consists of

   - 90 rolls of width 130 cm,
   - 610 rolls of width 108 cm,
   - 395 rolls of width 42 cm, and
   - 211 rolls of width 93 cm.

   Formulate an integer linear program that minimizes the number of produced 3 m rolls and allows a correct cutting of the ordered rolls. (4 points)

2. Show that the dimension of a non-empty set $X \subseteq \mathbb{R}^n$ is the largest d for which X contains elements $v_0, v_1, \ldots, v_d$ such that $v_1 - v_0, v_2 - v_0, \ldots, v_d - v_0$ are linearly independent. Moreover, prove that a polyhedron $X \subseteq \mathbb{R}^n$ is of dimension n if and only if X contains a point in its interior. (5 points)

3. (a) Prove that hyperplanes, half-spaces, and polyhedra are closed and convex.

   (b) Prove that $\text{conv}\{e_1, \ldots, e_n\} = \{x \in \mathbb{R}^n \ : \ x \geq 0, \sum_{j=1}^n x_j = 1\}$ where e_j is the j-th unit vector ($j = 1 \ldots, n$). (4+4 points)

4. Let $X \subseteq \mathbb{R}^n$.

   (a) Prove that $\text{conv}(X)$ is the smallest convex set containing X.

   (b) Show that if X is finite, then $\text{conv}(X)$ is compact. (4+4 points)

Due date: Tuesday, November 3, 2015, before the lecture.