Winter term 2015/16 Research Institute for Discrete Mathematics
Dr. U. Brenner University of Bonn

Linear and Integer Optimization
Assignment Sheet 2

1. Prove that any set $X \in \mathbb{R}^n$ with $|X| > n + 1$ can be decomposed into subsets X_1 and X_2 such that $\mathrm{conv}(X_1) \cap \mathrm{conv}(X_2) \neq \emptyset$. (4 points)

2. (a) Prove the generalized Farkas Lemma (Theorem 5 in the lecture notes).
   (b) Let (P) be a linear program of the form $\min\{c^t x \mid Ax \leq b\}$. Show that the dual of the dual of (P) is equivalent to (P). (2+3 Points)

3. Consider the following linear program $\min\{c^t x \mid Ax = b\}$. Show that it either does not have a solution, it is unbounded, or all feasible solutions are optimal. Does this statement hold if we additionally require $x \geq 0$? (3 points)
   Hint: Consider the dual LP.

4. Let P be a polyhedron. Show that the problem of finding the largest ball that can be contained in P can be written as a linear program. (3 points)

Due date for the first 4 exercises: Tuesday, November 10, 2015, before the lecture.

p.t.o.

5. **Programming Exercise 1**
Implement the Fourier-Motzkin Elimination to decide if an LP $\max\{c^t x \mid Ax \leq b\}$ has a feasible solution. If it has a solution, print a solution vector to the standard output as a single line. If it does not have a solution, print the string "empty" followed by a certificate vector according to Farkas' Lemma (Theorem 4 of the lecture notes). The program has to be implemented in C/C++ using the GNU compilers gcc or g++. The program should be run from the command line and read in a text file, whose name is given as an argument. The text file specifies the LP in the following format:

- The first line contains the number m of rows and n of columns of A.
- The second line contains n floating point numbers specifying c.
- The third line contains m floating point numbers specifying b.
- The next m lines contain the rows of A. Each line contains the n floating point numbers in the respective row.

Example: The linear program

$$\max(-2, 0, 8) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$
$$\begin{pmatrix} 3.5 & -2 & 5 \\ 0 & 1 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \leq \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

would be encoded as:

```
2 3
-2.0   0.0   8.0
3.0   0.0
3.5   -2.0   5.0
0.0   1.0   -4.0
```

Of course, you can ignore the objective function because in this programming exercise we only want to check feasibility.

On the web site to the exercises you find test instances and an example program in C for reading the input. You may use the example as a base for your implementation. (10 points)

Due date for the programming exercise: Tuesday, November 24, 2015, before the lecture. Please send your solution by e-mail to your tutor.