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Exercise Set 5

Exercise 5.1. Recall exercise 2.2. Although you only had to solve parts (vi) and (vii) for bipartite graphs, they can also be solved in general for undirected graphs (with the same runtime), only with a more complicated algorithm in part (vi). Using this, describe a linear-time approximation scheme for the Maximum Cardinality Matching Problem. More specifically, describe an algorithm which takes as input an undirected graph G and a positive number ε, outputs a matching M in G such that $|M| \geq (1-\varepsilon) \cdot \nu(G)$, and runs in

$$O(\frac{1}{\varepsilon} \cdot (|V(G)| + |E(G)|)\,)\text{-time.}$$

(4 points)

Exercise 5.2. Let $G = (V, E)$ be an undirected graph and Q its fractional perfect matching polytope, which is defined by

$$Q = \{x \in \mathbb{R}^E : x_e \geq 0 \ (e \in E), \sum_{e \in \delta(v)} x_e = 1 \ (v \in V)\}.$$

Prove that a vector $x \in Q$ is a vertex of Q if and only if there exist vertex disjoint odd circuits $C_1, \ldots, C_k$ and a perfect matching M in $G - (V(C_1) \cup \ldots \cup V(C_k))$ such that

$$x_e = \begin{cases} \frac{1}{2} & \text{if } e \in E(C_1) \cup \ldots \cup E(C_k), \\ 1 & \text{if } e \in M, \\ 0 & \text{otherwise.} \end{cases}$$

(4 points)

Exercise 5.3. Let $k \in \mathbb{N}$, $k \geq 1$, and suppose G is a k-regular and $(k-1)$-edge-connected graph with an even number of vertices, and with edge weights $c : E(G) \to \mathbb{R}$. Show that there is a perfect matching M in G with $c(M) \leq (1/k) \cdot c(E(G))$.

(4 points)

Exercise 5.4. Let $n \in \mathbb{N}$. A graph with $2n+1$ vertices is called a *double star* if it emerges from a star with $n+1$ vertices by replacing every edge $\{v, w\}$ by a vertex z_{vw} and two edges $\{v, z_{vw}\}, \{z_{vw}, w\}$.
Show that there exists a polynomial time algorithm that, given a cost function c on the edges of the complete graph K_{2n+1}, finds a spanning double star S of K_{2n+1} that minimizes $c(E(S))$.

(4 points)

Special announcement: The student council of mathematics will organize the math party on 23.11. in N8schicht. The presale will be held on 20.11. (Monday), 21.11. (Tuesday) and 22.11. (Wednesday) in the mensa Poppelsdorf. Further information can be found at `fsmath.uni-bonn.de`.

Deadline: November 16th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws17/co_exercises/exercises.html`

In case of any questions feel free to contact me at silvanus@or.uni-bonn.de.