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Exercise Set 11

Exercise 11.1. Let G be an undirected graph. Given a partition $(X_1, \dots, X_k)$ of $V(G)$ we define $\delta(X_1, \dots, X_k) := \delta(X_1) \cup \dots \cup \delta(X_k)$ (so, in particular, if $\emptyset \neq X \subsetneq V(G)$ we have $\delta(X) = \delta(X, V(G) \setminus X)$). Consider the polytope

$$R_G := \Bigg\{ x : E(G) \to [0,1] \; : \sum_{e \in E(G)} x(e) = |V(G)| - 1 \text{ and } \sum_{e \in \delta(X_1,\dots,X_k)} x(e) \geq k - 1 \text{ for every partition } (X_1, \dots, X_k) \text{ of } V(G) \Bigg\}$$

(Compare with exercise 7.5.)
Show that R_G is the spanning-tree polytope of G.

(4 points)

Exercise 11.2. Consider the Nearest Neighbor Heuristic (NNH): Given an instance (K_n, c) of the TSP, choose some $v_1 \in V(K_n)$. For $i = 2, \dots, n$, choose $v_i \in V(K_n) \setminus \{v_1, \dots, v_{i-1}\}$ such that $c(\{v_{i-1}, v_i\})$ is smallest possible. Return the tour given by the vertex sequence $(v_1, \dots, v_n)$. Denote by $\text{opt}^{NNH}(K_n, c)$ the shortest possible length of any tour returned by the Nearest Neighbor Heuristic (i.e., taking the minimum over all possible choices within the algorithm), and by $\text{opt}(K_n, c)$ the length of an optimum tour. Show that the ratio $\text{opt}^{NNH}(K_n, c)/\text{opt}(K_n, c)$ can be arbitrarily large.

(4 points)

Exercise 11.3. Given an instance (K_n, c) of the TSP, denote by $HK(K_n, c)$ the Held-Karp lower bound and by $\text{opt}(K_n, c)$ the length of an optimum tour. Show that for instances of the Metric TSP the ratio $\text{opt}(K_n, c)/HK(K_n, c)$ can be arbitrarily close to $4/3$.

(5 points)

Exercise 11.4. We consider the Another Hamiltonian Circuit Problem as defined in exercise 10.3.

Show that the Another Hamiltonian Circuit Problem, restricted to simple graphs G with $|\delta(v)|$ odd for all $v \in V(G)$, is in P.

(3 points)

Deadline: January 11th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws17/co_exercises/exercises.html`

In case of any questions feel free to contact me at silvanus@or.uni-bonn.de.