Exercise Set 5

Exercise 5.1. Let $n \in \mathbb{N}$. A graph with $2n+1$ vertices is called a *double star* if it emerges from a star with $n+1$ vertices by replacing every edge $\{v, w\}$ by a vertex z_{vw} and two edges $\{v, z_{vw}\}, \{z_{vw}, w\}$.
Show that there exists a polynomial time algorithm that, given a cost function c on the edges of the complete graph K_{2n+1}, finds a spanning double star S of K_{2n+1} that minimizes $c(E(S))$.

(4 points)

Exercise 5.2. Let $k \in \mathbb{N}$, $k \geq 1$, and suppose G is a k-regular and $(k-1)$-edge-connected graph with an even number of vertices, and with edge weights $c : E(G) \to \mathbb{R}$. Show that there is a perfect matching M in G with $c(M) \leq (1/k) \cdot c(E(G))$.

(5 points)

Exercise 5.3. Let $G = (V, E)$ be an undirected graph and Q its fractional perfect matching polytope, which is defined by

$$Q = \{x \in \mathbb{R}^E : x_e \geq 0 \ (e \in E), \sum_{e \in \delta(v)} x_e = 1 \ (v \in V)\}.$$

Prove that a vector $x \in Q$ is a vertex of Q if and only if there exist vertex disjoint odd circuits $C_1, \dots, C_k$ and a perfect matching M in $G - (V(C_1) \cup \dots \cup V(C_k))$ such that

$$x_e = \begin{cases} \frac{1}{2} & \text{if } e \in E(C_1) \cup \dots \cup E(C_k), \\ 1 & \text{if } e \in M, \\ 0 & \text{otherwise.} \end{cases}$$

(6 points)

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Exercise 5.4. Given an undirected graph G and disjoint sets $S_e, S_o \subseteq V(G)$, a *partial* (S_e, S_o)*-join* is a set $J \subseteq E(G)$ such that $|\delta(v) \cap J|$ is even for every $v \in S_e$ and odd for every $v \in S_o$. (In particular, a T-join is the same as a partial $(V(G) \setminus T, T)$-join.) Consider the Minimum Weight Partial (S_e, S_o)-Join Problem: Given an undirected graph G with edge-weights $c : E(G) \to \mathbb{R}_{\geq 0}$ and disjoint sets $S_e, S_o \subseteq V(G)$, find a partial (S_e, S_o)-join of minimum weight, or determine that none exists. Show that this problem can be linearly reduced to the Minimum Weight T-Join Problem.

(5 points)

Deadline: November 14^{th}, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws19/co_exercises/exercises.html`

In case of any questions feel free to contact me at rabenstein@or.uni-bonn.de.