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Exercise Set 8

Exercise 8.1. Let G be a planar graph with capacities $u : E(G) \to \mathbb{R}_+$. Give an algorithm running in polynomial time that computes a set $C \subseteq V(G)$ such $\sum_{e \in \delta(C)} u(e)$ is maximal.

Hint: Max-Cut in general graphs is NP-hard, so planarity needs to be exploited. In particular try to describe what cuts in G correspond to in the planar dual of G.

(6 points)

Exercise 8.2. Let G be a connected graph, $c : E(G) \to \mathbb{R}$ and $T \subseteq V(G)$. Note that $|T|$ may be either even or odd. Consider the problem of finding a T^*-join J^* with $|T^* \Delta T| \leq 2$ such that $c(J^*)$ is minimum among all such J^*. Show that this problem can be linearly reduced to the Minimum Weight T-Join Problem.

(4 points)

Exercise 8.3. Consider the Variable Endpoints Path TSP: given a complete graph G with a metric cost function $d : E(G) \to \mathbb{R}_{\geq 0}$, compute a minimum-cost Hamiltonian path in G. The start and end points of the path are not specified in the input. Give a polynomial time algorithm that computes a 3/2 approximation for this problem using the result from Exercise 8.2.

(4 points)

Exercise 8.4. Describe a set of instances of the Metric TSP for which Christofides' Algorithm returns a tour whose length is arbitrarily close to 3/2 times the optimum.

(4 points)

Deadline: Dec 3rd, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws24/co_exercises_ws.html`

In case of any questions feel free to contact me at mkaul@uni-bonn.de.